

From Geometric Albedo to Bond Albedo: What Does an Albedo Actually Measure?

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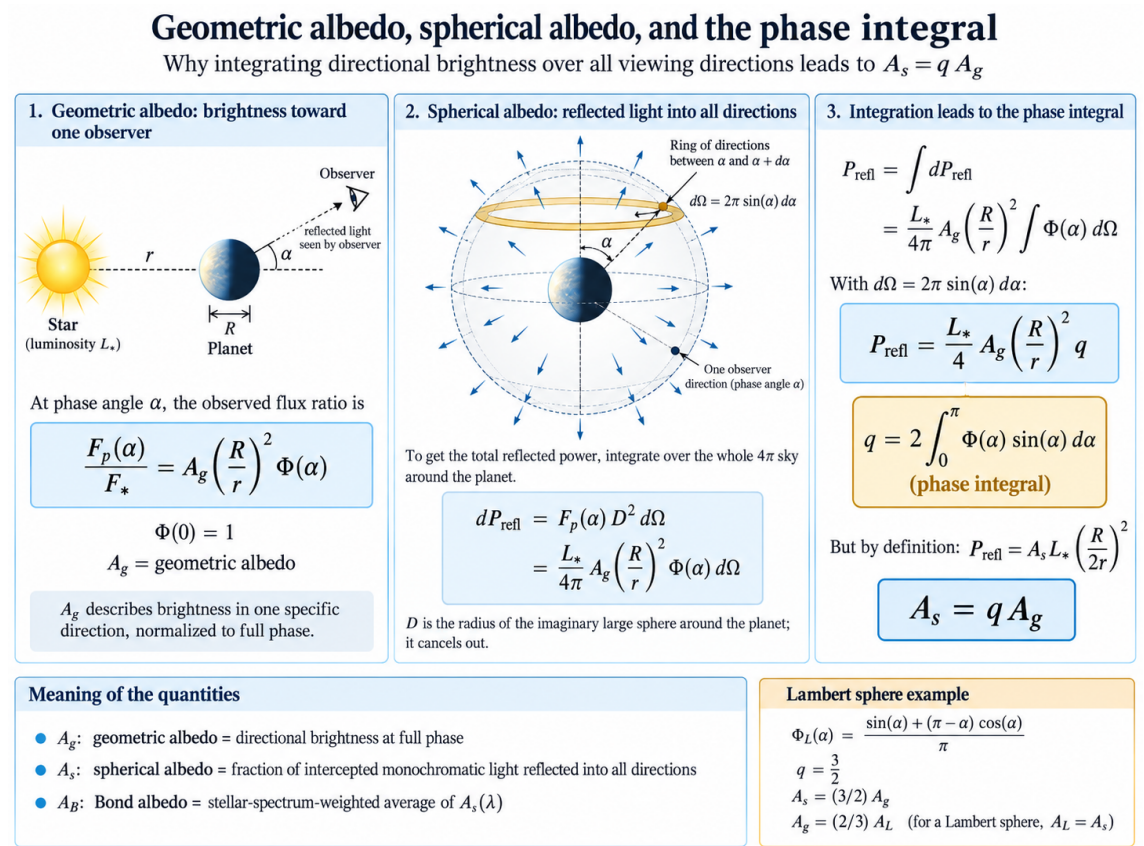


Figure 1: Illustration of the relationships between geometric and spherical albedos and Bond albedo. Figure was made by ChatGPT.

1 Introduction

The word albedo is used for several quantities that are closely related but are not interchangeable. This distinction is particularly important in exoplanet phase-curve studies. A secondary-eclipse measurement constrains the brightness of the planet in essentially one direction, whereas planetary energy balance depends on the total power reflected into all directions and over all wavelengths. Confusing these quantities can introduce factors of order unity—or even much larger errors when a broad passband or thermal emission is involved.

The natural starting point for an observer is the geometric albedo.

2 Geometric albedo and the observed phase curve

Consider a spherical planet of radius R_p at star–planet separation r . Let α denote the phase angle: $\alpha = 0$ corresponds to full phase, while $\alpha = \pi$ corresponds to the observer looking at the unilluminated hemisphere (cf. Figure 1).

The reflected-light planet-to-star flux ratio is conventionally written

$$\frac{F_p(\alpha, \lambda)}{F_\star(\lambda)} = A_g(\lambda) \left(\frac{R_p}{r} \right)^2 \Phi(\alpha, \lambda),$$

where

$$\Phi(0, \lambda) = 1.$$

Here $A_g(\lambda)$ is the geometric albedo, λ is the wavelength at which the flux ratio is evaluated, and Φ is the normalized phase function.

At full-planet phase,

$$\left. \frac{F_p}{F_\star} \right|_{\alpha=0} = A_g(\lambda) \left(\frac{R_p}{r} \right)^2.$$

Thus the geometric albedo is a directional brightness quantity. Historically, it is defined by comparing the full-phase brightness of the spherical object with that of a perfectly reflecting flat Lambert disk of the same cross-sectional area at the same distance.

This definition is important: A_g does not directly say what fraction of all incident stellar energy is reflected.

3 From one observer to all observing directions: spherical albedo

In the following derivation the wavelength dependence is temporarily suppressed for clarity; all quantities may be understood monochromatically.

To obtain the total reflected power, imagine a very large sphere of radius D surrounding the planet. Every point on this imaginary sphere represents a possible observing direction.

The stellar flux at distance D is

$$F_{\star} = \frac{L_{\star}}{4\pi D^2}.$$

The planetary flux in a direction corresponding to phase angle α is therefore

$$F_p(\alpha) = \frac{L_{\star}}{4\pi D^2} A_g \left(\frac{R_p}{r} \right)^2 \Phi(\alpha).$$

A small element of solid angle $d\Omega$ corresponds to area

$$dS = D^2 d\Omega$$

on our imaginary sphere. The reflected power crossing this surface element is

$$dP_{\text{refl}} = F_p(\alpha) D^2 d\Omega.$$

Substitution gives

$$dP_{\text{refl}} = \frac{L_{\star}}{4\pi D^2} A_g \left(\frac{R_p}{r} \right)^2 \Phi(\alpha) D^2 d\Omega.$$

thus (D cancels out):

$$dP_{\text{refl}} = \frac{L_{\star}}{4\pi} A_g \left(\frac{R_p}{r} \right)^2 \Phi(\alpha) d\Omega.$$

The artificial radius D has disappeared, as it must.

If the scattering pattern is axially symmetric around the star–planet axis,

$$d\Omega = \sin \alpha \, d\alpha \, d\varphi.$$

Integration over azimuth gives

$$\int_0^{2\pi} d\varphi = 2\pi,$$

and therefore

$$P_{\text{refl}} = \frac{L_{\star}}{2} A_g \left(\frac{R_p}{r} \right)^2 \int_0^{\pi} \Phi(\alpha) \sin \alpha d\alpha.$$

This immediately motivates the definition of the phase integral q :

$$q(\lambda) = 2 \int_0^{\pi} \Phi(\alpha, \lambda) \sin \alpha d\alpha$$

Thus

$$P_{\text{refl}} = \frac{L_{\star}}{4} A_g(\lambda) q(\lambda) \left(\frac{R_p}{r} \right)^2.$$

The factor $\sin \alpha$ has a simple geometrical origin. Directions between α and $\alpha + d\alpha$ form a ring on the surrounding sphere whose solid angle is

$$d\Omega = 2\pi \sin \alpha d\alpha.$$

Consequently, phase angles around quadrature represent much more solid angle than directions very close to full or new phase.

4 Spherical albedo

The star supplies at the planet an irradiance

$$F_{\text{inc}} = \frac{L_{\star}}{4\pi r^2}.$$

The planet intercepts only its projected cross section,

$$\pi R_p^2.$$

Therefore the intercepted power is

$$P_{\text{inc}} = \pi R_p^2 F_{\text{inc}} = \frac{L_{\star}}{4} \left(\frac{R_p}{r} \right)^2 = L_{\star} \left(\frac{R_p}{2r} \right)^2.$$

This is the origin of the frequently encountered $R_p/(2r)$ factor: it comes from the ratio

$$\frac{\pi R_p^2}{4\pi r^2}.$$

The spherical albedo is defined as the fraction of this monochromatic incident power that is scattered back into space, integrated over all outgoing directions:

$$A_s(\lambda) = \frac{P_{\text{refl}}(\lambda)}{P_{\text{inc}}(\lambda)}.$$

Substituting the expressions above yields

$$A_s(\lambda) = \frac{\frac{1}{4}L_\star A_g(\lambda)q(\lambda)(R_p/r)^2}{\frac{1}{4}L_\star(R_p/r)^2},$$

and therefore

$$A_s(\lambda) = q(\lambda)A_g(\lambda)$$

The relation is not an additional physical assumption. It follows simply from integrating the directional brightness represented by the geometric albedo and phase function over the entire sky.

5 The Lambert sphere

The most useful benchmark is an ideal Lambertian sphere. Before discussing the sphere itself, it is useful to make explicit the reference object that appears in the definition of geometric albedo.

The reference Lambert disk

The reference object is a flat, perfectly reflecting Lambertian disk of radius R_p , placed at the same distance from the star as the planet and viewed face-on. A Lambertian surface has direction-independent radiance. If its diffuse surface reflectivity is A_L , and the incident irradiance normal to the surface is F_{inc} , its radiance is

$$I_L = \frac{A_L F_{\text{inc}}}{\pi}.$$

For the perfectly reflecting reference disk, $A_L = 1$, and hence

$$I_{\text{disk}} = \frac{F_{\text{inc}}}{\pi}.$$

If the observer is at distance D , the disk is seen face-on and subtends the projected area πR_p^2 . The received flux is therefore

$$F_{\text{disk}} = \frac{1}{D^2} \int_{\text{disk}} I_{\text{disk}} dA = \frac{F_{\text{inc}}}{\pi D^2} \pi R_p^2 = F_{\text{inc}} \frac{R_p^2}{D^2}.$$

Since

$$F_{\text{inc}} = \frac{L_{\star}}{4\pi r^2} \quad \text{and} \quad F_{\star} = \frac{L_{\star}}{4\pi D^2},$$

we obtain

$$\boxed{\frac{F_{\text{disk}}}{F_{\star}} = \left(\frac{R_p}{r}\right)^2}.$$

The geometric albedo is defined by comparing the full-phase brightness of the actual object with this reference disk:

$$\boxed{A_g = \frac{F_p(0)}{F_{\text{disk}}}}.$$

Thus $A_g = 1$ means that at full phase the object is exactly as bright as a perfectly reflecting flat Lambert disk of the same radius. It does not mean that the object reflects 100% of the intercepted stellar energy.

The Lambert sphere at full phase

For a Lambertian sphere the local incident irradiance is not constant over the illuminated hemisphere. If θ is the angle measured from the substellar point and $\mu = \cos \theta$, the incident irradiance on a surface element is $F_{\text{inc}}\mu$. The local Lambertian radiance is therefore

$$I(\mu) = \frac{A_L F_{\text{inc}} \mu}{\pi}.$$

At full phase the illumination and viewing directions coincide, so the projected contribution of the same surface element toward the observer introduces another factor μ :

$$dF_p = \frac{I(\mu)\mu dA}{D^2}.$$

Hence

$$F_p(0) = \frac{A_L F_{\text{inc}}}{\pi D^2} \int_{\text{illuminated hemisphere}} \mu^2 dA.$$

For a sphere,

$$dA = R_p^2 \sin \theta d\theta d\varphi,$$

and therefore

$$\int \mu^2 dA = 2\pi R_p^2 \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta = \frac{2\pi R_p^2}{3}.$$

Thus

$$F_p(0) = \frac{2}{3} A_L F_{\text{inc}} \frac{R_p^2}{D^2} = \frac{2}{3} A_L F_{\text{disk}},$$

and consequently

$$\boxed{A_g = \frac{2}{3} A_L}.$$

The normalized phase function of a Lambert sphere is

$$\Phi_L(\alpha) = \frac{\sin \alpha + (\pi - \alpha) \cos \alpha}{\pi},$$

with $\Phi_L(0) = 1$. This normalization means only that full phase is used as the reference brightness of the phase function:

$$\Phi_L(\alpha) = \frac{F_p(\alpha)}{F_p(0)}.$$

It does *not* mean that all reflected power is directed toward the observer at $\alpha = 0$. The absolute full-phase flux is set by A_g :

$$\left. \frac{F_p}{F_\star} \right|_{\alpha=0} = A_g \left(\frac{R_p}{r} \right)^2.$$

For a perfectly reflecting Lambert sphere, $A_L = 1$, this becomes

$$\left. \frac{F_p}{F_\star} \right|_{\alpha=0} = \frac{2}{3} \left(\frac{R_p}{r} \right)^2.$$

The phase integral of the Lambert phase function is

$$q_L = 2 \int_0^\pi \Phi_L(\alpha) \sin \alpha \, d\alpha = \frac{3}{2}.$$

Hence

$$A_s = \frac{3}{2} A_g,$$

or

$$A_g = \frac{2}{3} A_s.$$

For an ideal Lambert surface, the fraction of incident power diffusely reflected is A_L , so for a Lambert sphere

$$A_s = A_L.$$

A perfectly reflecting Lambert sphere therefore has

$$A_s = A_L = 1, \quad A_g = \frac{2}{3}.$$

There is no contradiction. $A_s = 1$ states that all intercepted radiation is returned to space, whereas $A_g = 2/3$ states that the full-phase brightness of the sphere is two-thirds of the brightness of the perfectly reflecting flat Lambert disk used as the reference in the definition of geometric albedo. It does not mean that an observer receives two-thirds of the planet's total reflected power.

Different phase functions imply different phase integrals. There is therefore no universal conversion factor between A_g and A_s . Heng, Morris & Kitzmann showed explicitly how geometric albedo, phase-curve shape, phase integral and spherical albedo can be treated self-consistently for general scattering laws.

6 From spherical albedo to Bond albedo

The spherical albedo remains wavelength-dependent. The Bond albedo (named after US astronomer George Phillips Bond (1825–1865)) removes the wavelength dependence by integrating over the spectrum of the illuminating star:

$$A_B = \frac{\text{all reflected bolometric energy}}{\text{all incoming bolometric energy}}.$$

so, mathematically, it is

$$A_B = \frac{\int_0^\infty A_s(\lambda) F_{\star,\lambda} d\lambda}{\int_0^\infty F_{\star,\lambda} d\lambda}.$$

Using $A_s = qA_g$,

$$A_B = \frac{\int_0^\infty q(\lambda) A_g(\lambda) F_{\star,\lambda} d\lambda}{\int_0^\infty F_{\star,\lambda} d\lambda}.$$

The Bond albedo is therefore not a property of the planet alone. Even if the planet has a fixed wavelength-dependent reflectance spectrum, its Bond albedo depends on the spectral energy distribution of the illuminating star.

An atmosphere with strong blue scattering may therefore have a different Bond albedo around an F star than around a K star, even if its intrinsic wavelength-dependent scattering properties are unchanged.

7 Passband albedos

A telescope such as Kepler, TESS, CHEOPS or PLATO does not measure a monochromatic geometric albedo. It measures photons integrated through an instrumental response function.

Let $E_\nu(\nu)$ be the incident spectral energy distribution per unit frequency and let $S_X(\nu)$ be the dimensionless system throughput in passband X . Since one photon of frequency ν has energy $h\nu$, the detected photon rate in the frequency interval $d\nu$ is proportional to

$$dN_X(\nu) = S_X(\nu) \frac{E_\nu(\nu)}{h\nu} d\nu.$$

For wavelength and frequency,

$$\nu = \frac{c}{\lambda}, \quad \left| \frac{d\nu}{d\lambda} \right| = \frac{c}{\lambda^2}.$$

The spectral densities E_ν and E_λ refer to the same energy contained in corresponding spectral intervals, so

$$E_\nu(\nu) |d\nu| = E_\lambda(\lambda) d\lambda.$$

Consequently,

$$E_\lambda(\lambda) = E_\nu(\nu) \left| \frac{d\nu}{d\lambda} \right| = E_\nu(\nu) \frac{c}{\lambda^2},$$

or equivalently

$$E_\nu(\nu) = E_\lambda(\lambda) \frac{\lambda^2}{c}.$$

The total detected photon rate in passband X can therefore be written either as

$$N_X = \int_0^\infty S_X(\nu) \frac{E_\nu(\nu)}{h\nu} d\nu$$

or, after transforming to wavelength, as

$$\boxed{N_X = \int_0^\infty S_X(\lambda) E_\lambda(\lambda) \frac{\lambda}{hc} d\lambda}.$$

The factor $\lambda/(hc)$ simply converts spectral energy into photon number because a photon at wavelength λ has energy hc/λ . This expression assumes that $S_X(\lambda)$ is a dimensionless photon-detection throughput. If a published response function has already been photon-weighted, the factor $\lambda/(hc)$ must not be applied a second time.

At full phase, $\Phi(0, \lambda) = 1$. The passband-integrated planet-to-star photon-count ratio is then

$$\left. \frac{N_{p,X}}{N_{\star,X}} \right|_{\alpha=0} = \left(\frac{R_p}{r} \right)^2 \frac{\int_0^\infty A_g(\lambda) F_{\star,\lambda} S_X(\lambda) \lambda d\lambda}{\int_0^\infty F_{\star,\lambda} S_X(\lambda) \lambda d\lambda}.$$

This motivates the passband geometric albedo

$$\boxed{A_{g,X} = \left(\frac{r}{R_p} \right)^2 \left. \frac{N_{p,X}}{N_{\star,X}} \right|_{\alpha=0}}$$

and hence

$$A_{g,X} = \frac{\int_0^\infty A_g(\lambda) F_{\star,\lambda} S_X(\lambda) \lambda d\lambda}{\int_0^\infty F_{\star,\lambda} S_X(\lambda) \lambda d\lambda}.$$

This expression applies directly at full phase. If observations away from full phase are used to infer an albedo, the result can additionally depend on the phase function adopted in the light-curve model. In particular, if the true phase function is wavelength-dependent but the fit assumes a single $\Phi_{\text{adopted}}(\alpha)$, the fitted albedo becomes a model-dependent effective quantity rather than a pure passband average of $A_g(\lambda)$.

Consequently, an “optical albedo” is not a universal number. Kepler, TESS, CHEOPS and PLATO need not measure the same effective geometric albedo for the same atmosphere.

8 Thermal emission is not reflection

For sufficiently hot planets, the secondary-eclipse (occultation) depth contains both reflected and thermal contributions:

$$D_{\text{occ}} = \left(\frac{F_p}{F_\star} \right)_{\text{reflected}} + \left(\frac{F_p}{F_\star} \right)_{\text{thermal}}.$$

If the thermal component is ignored and the complete eclipse depth is interpreted through

$$\frac{F_p}{F_\star} = A_g \left(\frac{R_p}{r} \right)^2,$$

the inferred “geometric albedo” will be too large.

This becomes particularly important for ultra-hot Jupiters and red optical passbands such as TESS.

9 The hierarchy of albedos

The relationships can be summarized compactly as

$$A_g(\lambda) \xrightarrow{\times q(\lambda)} A_s(\lambda) \xrightarrow{\text{stellar-spectrum weighting}} A_B.$$

Geometric albedo is directional. Spherical albedo is an energy fraction at a specified wavelength. Bond albedo is the bolometric energy fraction.

Keeping these three levels separate helps us understand the physics, geometry and observable quantities involved in reflection-effect calculations.

A Lambertian scattering, Lambert surfaces, and geometric albedo larger than unity

The term “Lambertian” is used frequently in planetary photometry, but several related concepts are easily confused. In particular, a Lambertian surface is not an isotropic point scatterer, and a Lambertian planet does not radiate the same reflected power into every direction. This Appendix summarizes the relevant definitions and their relation to geometric and spherical albedo. For clarity, the wavelength dependence is suppressed below; all quantities may be understood monochromatically.

A.1 The local Lambert reflection law

Consider a small surface element dA illuminated by a collimated beam. Let F_0 denote the irradiance that would be received by a surface perpendicular to the incident beam, let i be the incidence angle measured from the local surface normal, and define

$$\mu_0 = \cos i.$$

The irradiance actually received by the tilted surface element is

$$F_{\text{inc,local}} = F_0 \mu_0.$$

An ideal Lambertian surface is defined by a reflected radiance that is independent of the outgoing direction. If A_L denotes the diffuse hemispherical reflectance (the Lambert surface albedo), then

$$I_{\text{refl}} = \frac{A_L}{\pi} F_0 \mu_0$$

for every outward direction above the surface. Equivalently, the bidirectional reflectance distribution function (BRDF) of an ideal Lambert surface is

$$f_r = \frac{A_L}{\pi}$$

with units of sr^{-1} . The reflected radiance is therefore independent of emission angle and azimuth for fixed illumination.

This should not be confused with isotropic emission of power. If e is the emission angle measured from the local surface normal and

$$\mu = \cos e,$$

then the power radiated by the surface element into solid angle $d\Omega$ is

$$dP = I_{\text{refl}} \mu dA d\Omega.$$

Thus the power per unit solid angle is proportional to $\mu = \cos e$. A Lambert surface has constant *radiance*, not constant power per steradian. An isotropic point source, by contrast, would have $dP/d\Omega = \text{const}$.

The normalization of the Lambert law follows by integrating over the outward hemisphere. Since

$$d\Omega = \sin e de d\varphi,$$

we obtain

$$\begin{aligned} F_{\text{refl}} &= \int_{2\pi} I_{\text{refl}} \mu d\Omega \\ &= 2\pi I_{\text{refl}} \int_0^{\pi/2} \cos e \sin e de \\ &= \pi I_{\text{refl}} \\ &= A_L F_0 \mu_0. \end{aligned}$$

Therefore

$$A_L = \frac{F_{\text{refl}}}{F_{\text{inc,local}}}$$

for an ideal Lambertian surface. A perfectly reflecting Lambert surface has $A_L = 1$; all locally incident power is then returned to the outward hemisphere, although it is distributed over many directions.

A.2 What is a Lambertian planet?

A Lambertian planet, or Lambert sphere, is an ideal spherical body for which every surface element obeys the Lambert reflection law above, usually with the same A_L over the surface. The planet itself may be geometrically spherical, but its reflected radiation field is not spherically symmetric because illumination by the star selects a preferred direction. For a homogeneous Lambert sphere

illuminated by a distant point source, the disk-integrated reflected light is axially symmetric around the star–planet axis and depends only on the phase angle α .

Only surface elements that are simultaneously illuminated ($\mu_0 > 0$) and visible to the observer ($\mu > 0$) contribute to the measured reflected flux. Disk integration yields the Lambert phase function

$$\Phi_L(\alpha) = \frac{\sin \alpha + (\pi - \alpha) \cos \alpha}{\pi}$$

with

$$\Phi_L(0) = 1.$$

The equality $\Phi_L(0) = 1$ is only a normalization to the planet’s own full-phase brightness. It does not mean that all reflected power reaches the observer at full phase.

The phase integral of a Lambert sphere is

$$q_L = 2 \int_0^\pi \Phi_L(\alpha) \sin \alpha \, d\alpha = \frac{3}{2}.$$

Since the local fraction of incident energy that is reflected is A_L , the global spherical albedo of a homogeneous Lambert sphere is

$$A_s = A_L.$$

Using $A_s = qA_g$ therefore gives

$$A_g = \frac{2}{3}A_L$$

and consequently

$$A_L = 1 \quad \implies \quad A_s = 1, \quad q_L = \frac{3}{2}, \quad A_g = \frac{2}{3}.$$

For a perfectly reflecting Lambert sphere the observable full-phase planet-to-star flux ratio is therefore

$$\left. \frac{F_p}{F_\star} \right|_{\alpha=0} = \frac{2}{3} \left(\frac{R_p}{r} \right)^2.$$

The factor $2/3$ does *not* mean that the observer receives two-thirds of the total intercepted stellar power. It means that, at full phase, the Lambert sphere is two-thirds as bright as the perfectly reflecting flat Lambert disk used as the reference object in the definition of geometric albedo.

A.3 Lambertian reflection versus a scattering phase function

In atmospheric radiative transfer it is useful to distinguish a Lambertian *surface boundary condition* from the microscopic scattering phase function of particles or molecules. A single-particle phase function $P(\Theta)$ describes the angular redistribution of a photon in one scattering event as a function of scattering angle Θ . Examples include isotropic scattering, Rayleigh scattering and strongly forward- or backward-scattering aerosol phase functions.

A Lambert surface, in contrast, specifies the emergent radiance after radiation interacts with a surface or an optically thick diffuse layer:

$$I_{\text{refl}} = \frac{A_L}{\pi} F_{\text{inc,local}}.$$

Thus “Lambertian scattering” in planetary light-curve work usually means that the emergent intensity follows this diffuse-reflection law. It should not be interpreted as a statement that individual particles scatter photons isotropically.

A.4 Can the geometric albedo exceed unity?

Yes, in general it can. Geometric albedo is a directional brightness ratio, not an energy fraction. It compares the object’s full-phase brightness with that of the perfectly reflecting flat Lambert disk. A non-Lambertian body that concentrates reflected radiation strongly toward the backscattering direction can therefore have

$$A_g > 1$$

without violating energy conservation.

The energy constraint applies instead to the spherical albedo:

$$A_s = qA_g \leq 1.$$

Consequently

$$\boxed{A_g \leq \frac{1}{q}.}$$

A geometric albedo larger than unity therefore requires a sufficiently small phase integral, i.e. a phase function strongly concentrated into a limited range of directions. Strong backscattering or a narrow opposition enhancement can produce such behaviour.

For an ideal Lambert sphere, however,

$$q_L = \frac{3}{2},$$

so energy conservation gives

$$A_g \leq \frac{2}{3}.$$

Hence a true Lambert sphere cannot have $A_g > 2/3$. Geometric albedos above unity are possible only for sufficiently non-Lambertian angular scattering distributions.

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